

Exercise 1F

$$1 \text{ a } \frac{1}{\sqrt{5}} = \frac{1 \times \sqrt{5}}{\sqrt{5} \times \sqrt{5}} \\ = \frac{\sqrt{5}}{5}$$

$$\text{b } \frac{1}{\sqrt{11}} = \frac{1 \times \sqrt{11}}{\sqrt{11} \times \sqrt{11}} \\ = \frac{\sqrt{11}}{11}$$

$$\text{c } \frac{1}{\sqrt{2}} = \frac{1 \times \sqrt{2}}{\sqrt{2} \times \sqrt{2}} \\ = \frac{\sqrt{2}}{2}$$

$$\text{d } \frac{\sqrt{3}}{\sqrt{15}} = \frac{\sqrt{3} \times \sqrt{15}}{\sqrt{15} \times \sqrt{15}} \\ = \frac{\sqrt{3 \times 15}}{15} \\ = \frac{\sqrt{45}}{15} \\ = \frac{\sqrt{9 \times 5}}{15} \\ = \frac{\sqrt{9} \times \sqrt{5}}{15} \\ = \frac{\sqrt{5}}{5}$$

$$\text{e } \frac{\sqrt{12}}{\sqrt{48}} = \frac{\sqrt{12}}{\sqrt{12} \times \sqrt{4}} \\ = \frac{1}{\sqrt{4}} \\ = \frac{1}{2}$$

$$\text{f } \frac{\sqrt{5}}{\sqrt{80}} = \frac{\sqrt{5}}{\sqrt{5} \times \sqrt{16}} \\ = \frac{1}{\sqrt{16}} \\ = \frac{1}{4}$$

$$\text{g } \frac{\sqrt{12}}{\sqrt{156}} = \frac{\sqrt{12}}{\sqrt{12} \times \sqrt{13}} \\ = \frac{1}{\sqrt{13}} \\ = \frac{1 \times \sqrt{13}}{\sqrt{13} \times \sqrt{13}} \\ = \frac{\sqrt{13}}{13}$$

$$\text{h } \frac{\sqrt{7}}{\sqrt{63}} = \frac{\sqrt{7}}{\sqrt{7} \times \sqrt{9}} \\ = \frac{1}{\sqrt{9}} \\ = \frac{1}{3}$$

$$2 \text{ a } \frac{1}{1 + \sqrt{3}} = \frac{(1 - \sqrt{3})}{(1 + \sqrt{3})(1 - \sqrt{3})} \\ = \frac{1 - \sqrt{3}}{1 - \sqrt{3} + \sqrt{3} - \sqrt{9}} \\ = \frac{-1 + \sqrt{3}}{2}$$

$$\text{b } \frac{1}{2 + \sqrt{5}} = \frac{(2 - \sqrt{5})}{(2 + \sqrt{5})(2 - \sqrt{5})} \\ = \frac{2 - \sqrt{5}}{4 - 5} \\ = \frac{2 - \sqrt{5}}{-1} \\ = -2 + \sqrt{5}$$

$$\text{c } \frac{1}{3 - \sqrt{7}} = \frac{3 + \sqrt{7}}{(3 - \sqrt{7})(3 + \sqrt{7})} \\ = \frac{3 + \sqrt{7}}{9 - 7} \\ = \frac{3 + \sqrt{7}}{2}$$

$$\begin{aligned}
 2 \text{ d } \frac{4}{3-\sqrt{5}} &= \frac{4 \times (3+\sqrt{5})}{(3-\sqrt{5})(3+\sqrt{5})} \\
 &= \frac{12+4\sqrt{5}}{9-5} \\
 &= \frac{12+4\sqrt{5}}{4} \\
 &= 3+\sqrt{5}
 \end{aligned}$$

$$\begin{aligned}
 \text{e } \frac{1}{\sqrt{5}-\sqrt{3}} &= \frac{\sqrt{5}+\sqrt{3}}{(\sqrt{5}-\sqrt{3})(\sqrt{5}+\sqrt{3})} \\
 &= \frac{\sqrt{5}+\sqrt{3}}{5-3} \\
 &= \frac{\sqrt{5}+\sqrt{3}}{2}
 \end{aligned}$$

$$\begin{aligned}
 \text{f } \frac{3-\sqrt{2}}{4-\sqrt{5}} &= \frac{(3-\sqrt{2})(4+\sqrt{5})}{(4-\sqrt{5})(4+\sqrt{5})} \\
 &= \frac{(3-\sqrt{2})(4+\sqrt{5})}{16-5} \\
 &= \frac{(3-\sqrt{2})(4+\sqrt{5})}{11} \\
 &= \frac{12+3\sqrt{5}-4\sqrt{2}-\sqrt{10}}{11}
 \end{aligned}$$

$$\begin{aligned}
 \text{g } \frac{5}{2+\sqrt{5}} &= \frac{5 \times (2-\sqrt{5})}{(2+\sqrt{5})(2-\sqrt{5})} \\
 &= \frac{5(2-\sqrt{5})}{4-5} \\
 &= \frac{5(2-\sqrt{5})}{-1} \\
 &= 5(\sqrt{5}-2)
 \end{aligned}$$

$$\begin{aligned}
 \text{h } \frac{5\sqrt{2}}{\sqrt{8}-\sqrt{7}} &= \frac{5\sqrt{2}(\sqrt{8}+\sqrt{7})}{(\sqrt{8}-\sqrt{7})(\sqrt{8}+\sqrt{7})} \\
 &= \frac{5(\sqrt{16}+\sqrt{14})}{8-7} \\
 &= 20+5\sqrt{14}
 \end{aligned}$$

$$\begin{aligned}
 \text{i } \frac{11}{3+\sqrt{11}} &= \frac{11(3-\sqrt{11})}{(3+\sqrt{11})(3-\sqrt{11})} \\
 &= \frac{11(3-\sqrt{11})}{9-11} \\
 &= \frac{11(3-\sqrt{11})}{-2} \\
 &= \frac{11(\sqrt{11}-3)}{2}
 \end{aligned}$$

$$\begin{aligned}
 \text{j } \frac{\sqrt{3}-\sqrt{7}}{\sqrt{3}+\sqrt{7}} &= \frac{(\sqrt{3}-\sqrt{7})(\sqrt{3}-\sqrt{7})}{(\sqrt{3}+\sqrt{7})(\sqrt{3}-\sqrt{7})} \\
 &= \frac{3-\sqrt{21}-\sqrt{21}+7}{3-7} \\
 &= \frac{10-2\sqrt{21}}{-4} \\
 &= \frac{\sqrt{21}-5}{2}
 \end{aligned}$$

$$\begin{aligned}
 \text{k } \frac{\sqrt{17}-\sqrt{11}}{\sqrt{17}+\sqrt{11}} &= \frac{(\sqrt{17}-\sqrt{11})(\sqrt{17}-\sqrt{11})}{(\sqrt{17}+\sqrt{11})(\sqrt{17}-\sqrt{11})} \\
 &= \frac{17-\sqrt{187}-\sqrt{187}+11}{17-11} \\
 &= \frac{28-2\sqrt{187}}{6} \\
 &= \frac{14-\sqrt{187}}{3}
 \end{aligned}$$

$$\begin{aligned}
 \text{l } \frac{\sqrt{41}+\sqrt{29}}{\sqrt{41}-\sqrt{29}} &= \frac{(\sqrt{41}+\sqrt{29})(\sqrt{41}+\sqrt{29})}{(\sqrt{41}-\sqrt{29})(\sqrt{41}+\sqrt{29})} \\
 &= \frac{41+2\sqrt{41}\sqrt{29}+29}{41-29} \\
 &= \frac{70+2\sqrt{1189}}{12} \\
 &= \frac{35+\sqrt{1189}}{6}
 \end{aligned}$$

$$\begin{aligned}
 2 \text{ m } \frac{\sqrt{2}-\sqrt{3}}{\sqrt{3}-\sqrt{2}} &= \frac{(\sqrt{2}-\sqrt{3})(\sqrt{3}+\sqrt{2})}{(\sqrt{3}-\sqrt{2})(\sqrt{3}+\sqrt{2})} \\
 &= \frac{\sqrt{6}+2-3-\sqrt{6}}{3-2} \\
 &= \frac{-1}{1} \\
 &= -1
 \end{aligned}$$

$$\begin{aligned}
 3 \text{ a } \frac{1}{(3-\sqrt{2})^2} &= \frac{1}{(3-\sqrt{2})(3-\sqrt{2})} \\
 &= \frac{1}{9-3\sqrt{2}-3\sqrt{2}+\sqrt{4}} \\
 &= \frac{1}{11-6\sqrt{2}} \\
 &= \frac{(11+6\sqrt{2})}{(11-6\sqrt{2})(11+6\sqrt{2})} \\
 &= \frac{11+6\sqrt{2}}{121+66\sqrt{2}-66\sqrt{2}-72} \\
 &= \frac{11+6\sqrt{2}}{49}
 \end{aligned}$$

$$\begin{aligned}
 \text{b } \frac{1}{(2+\sqrt{5})^2} &= \frac{1}{(2+\sqrt{5})(2+\sqrt{5})} \\
 &= \frac{1}{4+2\sqrt{5}+2\sqrt{5}+\sqrt{25}} \\
 &= \frac{1}{9+4\sqrt{5}} \\
 &= \frac{(9-4\sqrt{5})}{(9+4\sqrt{5})(9-4\sqrt{5})} \\
 &= \frac{9-4\sqrt{5}}{81-36\sqrt{5}+36\sqrt{5}-80} \\
 &= \frac{9-4\sqrt{5}}{1} \\
 &= 9-4\sqrt{5}
 \end{aligned}$$

$$\begin{aligned}
 3 \text{ c } \frac{4}{(3-\sqrt{2})^2} &= \frac{4}{(3-\sqrt{2})(3-\sqrt{2})} \\
 &= \frac{4}{9-3\sqrt{2}-3\sqrt{2}+\sqrt{4}} \\
 &= \frac{4}{11-6\sqrt{2}} \\
 &= \frac{4 \times (11+6\sqrt{2})}{(11-6\sqrt{2})(11+6\sqrt{2})} \\
 &= \frac{44+24\sqrt{2}}{121+66\sqrt{2}-66\sqrt{2}-72} \\
 &= \frac{44+24\sqrt{2}}{49}
 \end{aligned}$$

$$\begin{aligned}
 \text{d } \frac{3}{(5+\sqrt{2})^2} &= \frac{3}{(5+\sqrt{2})(5+\sqrt{2})} \\
 &= \frac{3}{25+5\sqrt{2}+5\sqrt{2}+\sqrt{4}} \\
 &= \frac{3}{27+10\sqrt{2}} \\
 &= \frac{3 \times (27-10\sqrt{2})}{(27+10\sqrt{2})(27-10\sqrt{2})} \\
 &= \frac{3 \times (27-10\sqrt{2})}{729-270\sqrt{2}+270\sqrt{2}-200} \\
 &= \frac{81-30\sqrt{2}}{529}
 \end{aligned}$$

$$\begin{aligned}
 \text{e } \frac{1}{(5+\sqrt{2})(3-\sqrt{2})} &= \frac{1}{15-5\sqrt{2}+3\sqrt{2}-\sqrt{4}} \\
 &= \frac{1}{13-2\sqrt{2}} \\
 &= \frac{(13+2\sqrt{2})}{(13-2\sqrt{2})(13+2\sqrt{2})} \\
 &= \frac{13+2\sqrt{2}}{169+26\sqrt{2}-26\sqrt{2}-8} \\
 &= \frac{13+2\sqrt{2}}{161}
 \end{aligned}$$

$$\begin{aligned}
 3 \text{ f } \frac{2}{(5-\sqrt{3})(2+\sqrt{3})} &= \frac{2}{10+5\sqrt{3}-2\sqrt{3}-\sqrt{9}} \\
 &= \frac{2}{7+3\sqrt{3}} \\
 &= \frac{2 \times (7-3\sqrt{3})}{(7+3\sqrt{3})(7-3\sqrt{3})} \\
 &= \frac{14-6\sqrt{3}}{49-21\sqrt{3}+21\sqrt{3}-27} \\
 &= \frac{14-6\sqrt{3}}{22} \\
 &= \frac{7-3\sqrt{3}}{11}
 \end{aligned}$$

$$\begin{aligned}
 4 \quad \frac{3-2\sqrt{5}}{\sqrt{5}-1} &= \frac{(3-2\sqrt{5})(\sqrt{5}+1)}{(\sqrt{5}-1)(\sqrt{5}+1)} \\
 &= \frac{3\sqrt{5}+3-10-2\sqrt{5}}{5+\sqrt{5}-\sqrt{5}-1} \\
 &= \frac{\sqrt{5}-7}{4} \\
 &= \frac{-7}{4} + \frac{\sqrt{5}}{4}
 \end{aligned}$$

$$p = -\frac{7}{4}, q = \frac{\sqrt{5}}{4}$$